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Temperature, transitivity, and the zeroth law

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Different statements of the zeroth law are examined. Two types of statements—which characterize two aspects of temperature—are found. A new formulation of the zeroth law is given and a corollary is stated. By means of this corollary it is shown how temperature and transitivity are used to disclose new nonthermal modes of interaction.

In this paper different forms of the zeroth law are compared. The aim is to show that these forms are not equivalent. Statements presented by Maxwell,[1] Sommerfeld,[2] and Thomsen[3] lead to a clarification of a problem which presents itself as follows. If an energy relation holds between the objects *A* and *B*, and between the objects *B* and *C*, does there then exist any criterion or axiom from which one can deduce that the relation also holds between *A* and *C*? Does there exist a guideline from which it follows that an energy relation is transitive?

The principle of transitivity has a fundamental position in mathematics and logic, as is well known. Mach was one of the first to draw attention to the role of this principle in physics; his definition of mass rests on it.[4] It is also involved in one of the most interesting problems in research. Redlich[5] presents it as follows:

> We choose an object *A* which is permeable for neutrons, an object *B* that absorbs neutrons, and an object *C* that radiates neutrons. Not knowing anything of these radiation properties, we establish thermal equilibrium between *A* and *B*, and between *A* and *C*. Then we find that *B* warms up on contact with *C*.
>
> Do we conclude that the concept of temperature is meaningless? By no means. We conclude that there is a new, nonthermal mode of interaction and set out to describe isolation and the particular conditions of interaction for this new mode.

The zeroth law is often presented in a form which expresses the transitivity of a certain relation between pairs of systems, viz. the relation of "being in diathermic equilibrium with." [6] The important point is that the notion of equality of temperature has a basic role in other forms from which the transitivity of an energy relation can be shown to follow.

DIFFERENT FORMS

It was Maxwell who first formulated a postulate which is still accepted as one form of the zeroth law.[7] He presented it as follows[1]: "Bodies whose temperatures are equal to that of the same body have themselves equal temperatures."

Caratheodory[8] and Born[9] were of the opinion that the definition of the concept of temperature requires, as an empirically based condition, the following form: "If two systems are both in thermal equilibrium with a third system, then they are in thermal equilibrium with each other." Fowler and Guggenheim[10] coined the name "zeroth law of thermodynamics" for this form.

Sommerfeld[2] preferred to take the notion of equality of temperature as a basic assumption. The following modification of his form has been proposed by Thomsen[3]: "There exists a scalar quantity called temperature which is a property of all thermodynamic systems (in equilibrium states), such that temperature equality is a necessary and sufficient condition for thermal equilibrium." Thomsen draws attention to the fact that the form, which expresses the transitivity of a relation—viz., the form which Caratheodory and Born used as a foundation in their pioneer papers[8,9]—follows immediately from his restatement of the zeroth law. From Maxwell[1] stems a similar statement: "If when two bodies are placed in thermal communication neither of them loses or gains heat, the two bodies are said to have equal temperatures or the same temperature. The two bodies are then said to be in thermal equilibrium."

THE NOTION OF TEMPERATURE EQUALITY AND EMPIRICAL TEMPERATURE

The transitivity of an energy relation between pairs of objects (systems) is commonly used to show that there exists a function of the state of an object which takes on the same value for all these objects[7,11,12]; as is known, this function is called empirical temperature. The transitivity condition can be arrived at through a principle which presents the notion of equality of temperature. It is important to distinguish between a principle which characterizes the concept of temperature in essence and a procedure which can be said to fulfill the requirement of such a principle; if this is not realized, great confusion may be the result.

Thomsen's restatement, for example, appears to present almost the essence of temperature (cf. below). However, another aspect of temperature results from any procedure which can be considered to fulfill the claim of a basic principle and which leads to empirical temperature; such a procedure may also be said to define temperature. It is probably most correct to say that temperature has two complementary aspects: viz., one which is implied in a basic principle and another which is implied in any process fulfilling such a principle.

It may be appropriate to elucidate this further by means of the following example: consider the concept of real numbers introduced by Dedekind.[13] According to him, a real number constitutes a means for dividing all rational numbers into two classes which have no element in common, but which together exhaust the whole domain of rational numbers. The important feature is that the essence of the concept of a real number is implied in the so-called Dedekind principle, viz., the following one: "If all rational numbers fall into two classes such that every number of the first class is less than every number of the second class, then there exists one and only one cut [*Schnitt*] which produces this division of all rational numbers into two classes."

Conversely, any process which is capable of effecting such a split in the domain of rational numbers is said to define a real number. Any scheme of classification which can be considered to be in accordance with the basic principle defines "an empirical" real number.[14]

Therefore, a real number, as characterized in essence by Dedekind's principle, is to "an empirical" real number as the concept of temperature, found in a basic principle, is to empirical temperature; precisely therein lies the difference between the statements above.

A BASIC PRINCIPLE

Watanabe[15] has drawn attention to the fundamental position which if–then relations have in science. Dedekind's principle is an if–then relation, and the intention is to express a basic zeroth law also as an if–then relation.

For that purpose, consider two objects, A and B, which we have strived to isolate with respect to all nonthermal modes of interaction except one. The mode is characterized by a generalized coordinate X and by a generalized force f.[5] The values, X_A, f_A and X_B, f_B, refer to A and B, respectively. The writer finds a basic (zeroth law) principle in the following statement: If the equilibrium values of X_A, f_A, X_B, and f_B cannot be varied independently but are related by the relationship

$$F_{A,B}(X_A, f_A, X_B, f_B) = 0,$$

then there exists one and only one quantity called temperature, which is a property of the state of each object, such that the equality of this quantity is a necessary and sufficient condition of the relationship.

TRANSITIVITY AND THE PRINCIPLE

The principle postulates that there exists a property of each object and, further, that the objects have the same quantity of this property. For three objects, A, B, and C, where, for example, A and B are separately in equilibrium with C, the principle asserts that the truth of the two relationships, $F_{A,C}$ and $F_{B,C}$, implies the truth of the third, $F_{A,B}$. It requires that the two relationships shall not be indiscriminately arbitrary functions of the variables.[13]

Thus the principle leads to the corollary: If, of three objects A, B, and C, A and B are separately in equilibrium with C, such that the relationships $F_{A,C}$ and $F_{B,C}$ fulfill the claim of the principle, then the relationship is transitive.

It is therefore enough to establish experimentally equilibrium between two pairs and to fulfill the claim of the principle for these pairs to be able to conclude that a definite relationship is transitive. The principle of transitivity plays a role in any process which leads to empirical temperature. The corollary—which, in fact, is equivalent to that form of the zeroth law which is most often presented—is indispensable for elucidating the role of transitivity and the concept of empirical temperature.

One may notice that Redlich's interpretation of the problem, which is cited above, is in accordance with these considerations. The point of importance for research is the following: if the principle of transitivity cannot be verified experimentally for three pairs, then one can conclude that there is a new type of nonthermal interaction.

The principle characterizes the concept of temperature in essence. This seems to be necessary but not sufficient. Empirical temperature is defined by any process which can be considered to fulfill the requirement of the principle and which leads to a quantity which can be associated with each object participating in the process, and any such process and the principle supplement each other. The principle constitutes a means for framing and checking the processes used; it characterizes their aim. Empirical temperatures are results of processes having a common aim.

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